IB MATHS AA HL · UNIT 2

Exponential and Logarithmic Functions: exam practice

10 questions · 40 marks · worked solutions and markschemes

Name _____ Date _____ Score / 40

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/exponential-and-logarithmic-functions-exam-practice

1. **3 marks · Paper 1 style · no calculator**

The graph of $y = ae^{bx}$ passes through $(0, 5)$ and $(2, 45)$, where $b > 0$.

Find a and the exact value of b. **[3]**

2. **4 marks · Paper 1 style · no calculator**

Solve $\log_3(2x + 1) - \log_3(x - 2) = 2$.

Find x. **[4]**

3. **4 marks · Paper 1 style · no calculator**

Solve $2\ln x - \ln(x + 6) = 0$.

Find x. **[4]**

4. **3 marks · Paper 1 style · no calculator**

The graph of $y = \log_2(x - 3) + 1$ is drawn.

(a) Write down the equation of the vertical asymptote. **[1]**

(b) Find the x-intercept. **[2]**

5. **3 marks · Paper 1 style · no calculator**

Let $g(x) = \ln(5 - x)$ for $x < 5$.

(a) Find $g^{-1}(x)$. **[2]**

(b) Write down the range of g^{-1}. **[1]**

6. **4 marks · Paper 2 style · calculator**

The value of a painting is modelled by $V = 2400 \times 1.06^t$ euros, t years after it was bought.

(a) Find the value after 12 years. **[2]**

(b) Find the value of t when the painting is first worth €6000. **[2]**

7. **4 marks · Paper 1 style · no calculator**

The graph of $y = 2^x$ is translated by $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ to give $y = f(x)$.

(a) Write down $f(x)$. **[1]**

(b) Find the x-intercept of the graph of f. **[2]**

(c) Write down the equation of the asymptote. **[1]**

8. **4 marks · Paper 2 style · calculator**

The loudness of a sound, L decibels, is $L = 10\log_{10}\left(\frac{I}{I_0}\right)$, where I is its intensity and I_0 is a fixed reference intensity.

(a) Find L when $I = 3.2 \times 10^6 I_0$. **[2]**

(b) Find the factor by which the intensity increases when L rises by 15 dB. **[2]**

9. **5 marks · Paper 1 style · no calculator**

Let $f(x) = 3e^{-x} + 2$.

(a) Write down the equation of the horizontal asymptote of the graph of f. **[1]**

(b) Find $f^{-1}(x)$ and state its domain. **[4]**

10. **6 marks · Paper 2 style · calculator**

The temperature of a cup of tea, $T\,°\text{C}$, t minutes after it is made is $T = 20 + 70e^{-0.08t}$.

(a) Find the temperature after 10 minutes. **[2]**

(b) Find the time for the tea to cool to $40\,°\text{C}$. **[3]**

(c) Write down the temperature the tea approaches in the long term. **[1]**

Answers and worked solutions

Question 1 — 3 marks

Answer: $a = 5, b = \ln 3$

$a = 5$ A1

$e^{2b} = 9$ M1

$b = \ln 3$ A1

Question 2 — 4 marks

Answer: $\frac{19}{7}$

$\log_3 \dfrac{2x+1}{x-2} = 2$ M1

$2x + 1 = 9(x - 2)$ A1

$7x = 19$ A1

$x = \dfrac{19}{7}$ A1

Question 3 — 4 marks

Answer: 3

$\ln \dfrac{x^2}{x+6} = 0$ M1

$x^2 = x + 6$ A1

$(x - 3)(x + 2) = 0$ A1

$x = 3$ ($x = -2$ rejected) A1

Question 4 — 3 marks

Answer: (a) $x = 3$; (b) $\frac{7}{2}$

(a)

$x = 3$ A1

(b)

$\log_2(x - 3) = -1$ M1

$x = \dfrac{7}{2}$ A1

Question 5 — 3 marks

Answer: (a) $g^{-1}(x) = 5 - e^x$; (b) $g^{-1}(x) < 5$

(a)

$5 - x = e^y$ M1

$g^{-1}(x) = 5 - e^x$ A1

(b)

$g^{-1}(x) < 5$ A1

Question 6 — 4 marks

Answer: (a) €4829.27; (b) $t = 15.7$

(a)

2400×1.06^{12} M1

€4829.27 A1

(b)

$1.06^t = 2.5$ M1

$t = 15.7$ A1

Question 7 — 4 marks

Answer: (a) $f(x) = 2^{x-3} - 4$; (b) 5; (c) $y = -4$

(a)

$f(x) = 2^{x-3} - 4$ A1

(b)

$2^{x-3} = 4$ M1

$x = 5$ A1

(c)

$y = -4$ A1

Question 8 — 4 marks

Answer: (a) 65.1 dB; (b) 31.6

(a)

$10 \log_{10}(3.2 \times 10^6)$ M1

65.1 dB A1

(b)

$10 \log_{10} k = 15$ M1

$k = 10^{1.5} = 31.6$ A1

Question 9 — 5 marks

Answer: (a) $y = 2$; (b) $f^{-1}(x) = \ln \dfrac{3}{x-2}$, $x > 2$

(a)

$y = 2$ A1

(b)

$y - 2 = 3e^{-x}$ M1

$x = -\ln \dfrac{y-2}{3}$ A1

$f^{-1}(x) = \ln \dfrac{3}{x-2}$ A1

$x > 2$ A1

Question 10 — 6 marks

Answer: (a) $51.5\,°\text{C}$; (b) 15.7 minutes; (c) $20\,°\text{C}$

(a)

$20 + 70e^{-0.8}$ M1

$51.5\,°\text{C}$ A1

(b)

$e^{-0.08t} = \dfrac{2}{7}$ M1

$t = \dfrac{\ln 3.5}{0.08}$ A1

15.7 min A1

(c)

$20\,°\text{C}$ A1

Every solution is also on ibmathrevision.com/hlaa/exponential-and-logarithmic-functions-exam-practice
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