IB MATHS AA HL · UNIT 4

Discrete Random Variables: exam practice

10 questions · 37 marks · worked solutions and markschemes

Name _____ Date _____ Score / 37

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/discrete-random-variables-exam-practice

1. **2 marks · Paper 2 style · calculator**

15% of seeds of a certain type fail to grow. A packet contains 25 seeds.

Find the probability that at most 2 seeds fail. **[2]**

2. **3 marks · Paper 1 style · no calculator**

A random variable X has $E(X) = 4$ and $\mathrm{Var}(X) = 3$.

Find $E(5 - 2X)$ and $\mathrm{Var}(5 - 2X)$. **[3]**

3. **3 marks · Paper 1 style · no calculator**

In a game a player rolls a fair six-sided dice. A 6 wins $\$10$, a 5 wins $\$4$, and any other score loses $\$3$.

Find the expected gain per game and state whether the game is fair. **[3]**

4. **4 marks · Paper 1 style · no calculator**

The random variable Y has distribution $P(Y=0)=0.2$, $P(Y=1)=0.5$, $P(Y=2)=0.3$.

Find $E(Y)$ and $\mathrm{Var}(Y)$. **[4]**

5. **4 marks · Paper 1 style · no calculator**

$X \sim B(n, p)$ with $E(X)=6$ and $\mathrm{Var}(X)=4.2$.

Find n and p. **[4]**

6. **3 marks · Paper 2 style · calculator**

$X \sim B(n,\ 0.4)$ and $P(X=0)<0.01$.

Find the least value of n. **[3]**

7. **4 marks · Paper 1 style · no calculator**

The random variable X has the distribution shown.

x	1	2	3	4
$P(X = x)$	0.1	k	0.3	$2k$

(a) Find k. **[2]**

(b) Find $E(X)$. **[2]**

8. **4 marks · Paper 2 style · calculator**

$X \sim B(12,\ 0.3)$.

(a) Find $P(X = 4)$. **[2]**

(b) Find $P(X \geq 3)$. **[2]**

9. **5 marks · Paper 2 style · calculator**

A multiple-choice test has 20 questions, each with 4 options. A student guesses every answer.

(a) Find the expected number of correct answers. **[1]**

(b) Find the probability that the student gets more than 8 correct. **[2]**

(c) Find the standard deviation of the number correct. **[2]**

10. **5 marks · Paper 1 style · no calculator**

Two fair four-sided dice, numbered 1 to 4, are rolled. X is the larger of the two scores.

(a) Show that $P(X = 3) = \frac{5}{16}$. **[2]**

(b) Find $E(X)$. **[3]**

Answers and worked solutions

Question 1 — 2 marks

Answer: 0.254

$X \sim B(25, 0.15)$, $P(X \leq 2)$ **M1**

0.254 **A1**

Question 2 — 3 marks

Answer: $E = -3$, $\text{Var} = 12$

$5 - 2(4) = -3$ **A1**

$(-2)^2 \times 3$ **M1**

12 **A1**

Question 3 — 3 marks

Answer: $\$0.33$; not fair

$\dfrac{10 + 4 - 3 \times 4}{6}$ **M1**

$\dfrac{1}{3}$ dollar **A1**

not fair ($E \neq 0$) **A1**

Question 4 — 4 marks

Answer: $E(Y) = 1.1$, $\text{Var}(Y) = 0.49$

$E(Y) = 1.1$ **A1**

$E(Y^2) = 0.5 + 1.2 = 1.7$ $M1$ **A1**

$\text{Var}(Y) = 1.7 - 1.21 = 0.49$ **A1**

Question 5 — 4 marks

Answer: $n = 20$, $p = 0.3$

$np = 6$, $np(1-p) = 4.2$ **M1**

$1 - p = 0.7$ **A1**

$p = 0.3$ **A1**

$n = 20$ **A1**

Question 6 — 3 marks

Answer: 10

$0.6^n < 0.01$ **M1**

$n > 9.02$ **A1**

$n = 10$ **A1**

Question 7 — 4 marks

Answer: (a) $\frac{1}{5}$; (b) 3

(a)

$0.4 + 3k = 1$ **M1**

$k = 0.2$ **A1**

(b)

$0.1 + 0.4 + 0.9 + 1.6$ **M1**

3 **A1**

Question 8 — 4 marks

Answer: (a) 0.231; (b) 0.747

(a)

$\binom{12}{4}0.3^4(0.7)^8$ **M1**

0.231 **A1**

(b)

$1 - P(X \leq 2)$ **M1**

0.747 **A1**

Question 9 — 5 marks

Answer: (a) 5; (b) 0.0409; (c) 1.94

(a)

$20 \times 0.25 = 5$ **A1**

(b)

$1 - P(X \leq 8)$ **M1**

0.0409 **A1**

(c)

$\sqrt{20(0.25)(0.75)}$ **M1**

1.94 **A1**

Question 10 — 5 marks

Answer: (b) $\frac{25}{8}$

(a)

outcomes $(3,1), (3,2), (3,3), (1,3), (2,3)$ **M1**

$\dfrac{5}{16}$ **A1 AG**

(b)

$P(X = x) = \dfrac{2x - 1}{16}$ **A1**

$\dfrac{1 + 6 + 15 + 28}{16}$ **M1**

$\dfrac{25}{8}$ **A1**

Every solution is also on ibmathrevision.com/hlaa/discrete-random-variables-exam-practice
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