Definite Integrals: exam practice

10 questions · 35 marks · worked solutions and markschemes

Name _____ Date _____ Score / 35

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/definite-integrals-exam-practice

1. **4 marks · Paper 1 style · no calculator**

Find the exact value of $\int_1^9 \left(2\sqrt{x} - \frac{3}{x^2}\right) dx$.

Evaluate the integral. **[4]**

2. **3 marks · Paper 1 style · no calculator**

Find the exact value of $\int_0^{\frac{\pi}{3}} \sin 2x \, dx$.

Evaluate the integral. **[3]**

3. **3 marks · Paper 2 style · calculator**

The region enclosed by $y = x^3 - 4x$ and the x-axis for $-2 \leq x \leq 2$ is shaded.

Find the total area of the shaded region. **[3]**

4. **3 marks · Paper 1 style · no calculator**

Find the exact value of $\int_0^{\ln 3} e^{2x}\,dx$.

Evaluate the integral. **[3]**

5. **3 marks · Paper 2 style · calculator**

Find the area enclosed by $y = \sin x$ and $y = \cos x$ between $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.

Find the area. **[3]**

6. **3 marks · Paper 1 style · no calculator**

Given that $\int_0^3 f(x)\,dx = 7$, find $\int_0^3 (2f(x) + x)dx$.

Find the value. **[3]**

7. **4 marks · Paper 1 style · no calculator**

Given that $\int_1^k (2x-1)\,dx = 12$ and $k > 1$, find k.

Find k. **[4]**

8. **4 marks · Paper 2 style · calculator**

Find the area enclosed by $y = 4x - x^2$ and $y = x$.

Find the area. **[4]**

9. **5 marks · Paper 1 style · no calculator**

Find the area enclosed between $y = x^2$ and $y = x + 6$.

Find the area. **[5]**

10. **3 marks · Paper 1 style · no calculator**

Find the exact value of $\int_1^e \frac{\ln x}{x}\,dx$.

Evaluate the integral. **[3]**

Answers and worked solutions

Question 1 — 4 marks

Answer: 32

$\left[\frac{4}{3}x^{\frac{3}{2}}+\frac{3}{x}\right]_1^9$ A1A1

$\left(36+\frac{1}{3}\right)-\left(\frac{4}{3}+3\right)$ M1

32 A1

Question 2 — 3 marks

Answer: $\frac{3}{4}$

$\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{3}}$ A1

$-\frac{1}{2}\left(-\frac{1}{2}\right)+\frac{1}{2}$ M1

$\frac{3}{4}$ A1

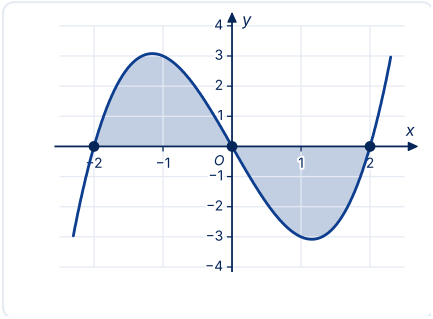
Question 3 — 3 marks

Answer: 8

roots $-2,\ 0,\ 2$; symmetric M1

$2\int_{-2}^{0}(x^3-4x)\,dx$ A1

8 A1



Two equal regions: area 8

Question 4 — 3 marks

Answer: 4

$\left[\frac{1}{2}e^{2x}\right]_0^{\ln 3}$ A1

$\frac{1}{2}(9-1)$ M1

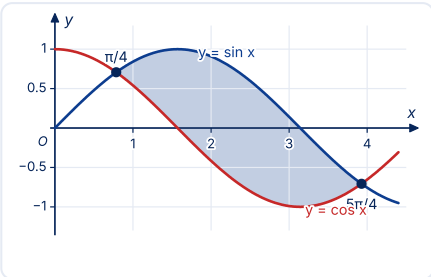
4 A1

Question 5 — 3 marks

Answer: $2\sqrt{2}$

$\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}}(\sin x-\cos x)\,dx$ M1A1

$2\sqrt{2}$ A1



Area $=2\sqrt{2}$

Question 6 — 3 marks

Answer: $\frac{37}{2}$

$2(7)+\int_0^3 x\,dx$ M1

$14+\frac{9}{2}$ A1

$\frac{37}{2}$ A1

Question 7 — 4 marks

Answer: 4

$\left[x^2-x\right]_1^k=k^2-k$ M1A1

$k^2-k-12=0$ A1

$k=4$ A1

Question 8 — 4 marks

Answer: $\frac{9}{2}$

meet at $x=0,\ x=3$ A1

$\int_0^3(3x-x^2)\,dx$ M1A1

$\frac{9}{2}$ A1

Question 9 — 5 marks

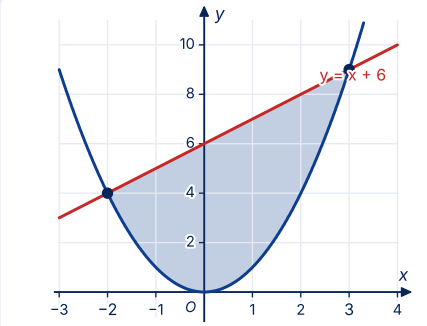
Answer: $\frac{125}{6}$

intersections $x=-2,\ x=3$ A1

$\int_{-2}^{3}(x+6-x^2)\,dx$ M1A1

$\left[\frac{x^2}{2}+6x-\frac{x^3}{3}\right]_{-2}^{3}$ A1

$\frac{125}{6}$ A1



Area $=\int$ from -2 to 3 of $(x+6-x^2)\,\mathrm{d}x$

Question 10 — 3 marks

Answer: $\frac{1}{2}$

$\left[\frac{1}{2}(\ln x)^2\right]_1^e$ M1A1

$\frac{1}{2}$ A1

Every solution is also on ibmathrevision.com/hlaa/definite-integrals-exam-practice
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