IB MATHS AA HL · UNIT 1

Counting and Binomials: exam practice

10 questions · 45 marks · worked solutions and markschemes

Name _____ Date _____ Score / 45

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/counting-and-binomials-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Find the number of different ways to arrange all the letters of the word **NUMBER** in a straight line.

2. **3 marks · Paper 2 style · calculator**

Find the coefficient of the term in x^3 in the binomial expansion of $(2-x)^8$.

3. **3 marks · Paper 1 style · no calculator**

A committee of 4 people is to be chosen from a group consisting of 6 men and 5 women. Find the number of different committees that can be formed if the committee must contain exactly 2 men and 2 women.

4. **4 marks · Paper 1 style · no calculator**

Find the number of different arrangements of the letters of the word $\mathrm{PARALLEL}$ in which the three letters L are together.

Find this number. **[4]**

5. **6 marks · Paper 2 style · calculator**

Consider the expansion of $(1-3x)^4(1-2kx)^2$, where k is a constant.

Given that the coefficient of the x^6 term is 36, find the possible values of k.

6. **6 marks · Paper 1 style · no calculator**

How many 5-digit numbers can be formed using the digits $1, 2, 3, 4, 5, 6, 7$ without repetition, if they must be even and strictly greater than 40000?

7. **4 marks · Paper 1 style · no calculator**

Seven people, including Ana and Ben, sit in a row of seven chairs.

(a) Find the number of arrangements in which Ana and Ben sit next to each other. **[2]**

(b) Find the number of arrangements in which Ana and Ben do not sit next to each other. **[2]**

8. **5 marks · Paper 2 style · calculator**

A committee of 5 is chosen from 6 teachers and 8 students.

(a) Find the number of committees with exactly 2 teachers. **[2]**

(b) Find the number of committees with at least one teacher. **[3]**

9. **7 marks · Paper 1 style · no calculator**

Consider the binomial expansion of $\left(\frac{ax}{2} + \frac{3}{x^2}\right)^5$.

1. Find an expression, in terms of a, for the coefficient of the x^{-1} term. **[5 marks]**
2. Given that the coefficient of the x^{-1} term is exactly 90, find the value of the constant a. **[2 marks]**

10. **5 marks · Paper 1 style · no calculator**

[AHL Extension]

1. Find the binomial expansion of $(1+3x)^{\frac{1}{2}}$ up to and including the term in x^2. **[2 marks]**
2. By choosing a suitable value for x, use your expansion from part (a) to find an estimate for $\sqrt{1.03}$. Give your final answer correct to 5 decimal places. **[3 marks]**

Answers and worked solutions

Question 1 — 2 marks

Answer: 720

All 6 letters of NUMBER are distinct, so use

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$ **(M1)**

720 **A1**

Question 2 — 3 marks

Answer: -1792

Term in x^3: $\binom{8}{3}(2)^5(-x)^3$ **(M1)**

Evaluate: $56 \times 32 \times (-1)$ **A1**

Coefficient $= -1792$ **A1**

Question 3 — 3 marks

Answer: 150

Choose 2 of 6 men and 2 of 5 women: $\binom{6}{2} \times \binom{5}{2}$ **(M1)**

$= 15 \times 10$ **(A1)**

$= 150$ **A1**

Question 4 — 4 marks

Answer: 360

treat LLL as one block: 6 items **M1**

two A's repeated **M1**

$\frac{6!}{2!}$ **A1**

360 **A1**

Question 5 — 6 marks

Answer: $-1/3, 1/3$

x^6 is the highest power: it comes only from the x^4 term of $(1-3x)^4$ times the x^2 term of $(1-2kx)^2$ **M1**

x^4 term: $(-3x)^4 = 81x^4$ **A1**

x^2 term: $(-2kx)^2 = 4k^2x^2$ **A1**

Equate the coefficient:

$81(4k^2) = 36$ **M1**

$324k^2 = 36 \Rightarrow k^2 = \frac{1}{9}$ **A1**

$k = \pm\frac{1}{3}$ **A1**

Question 6 — 6 marks

Answer: 600

The first digit must be $4, 5, 6$ or 7 and the last digit $2, 4$ or 6 **M1**

Split into cases by whether the first digit is odd or even (to avoid double-using an even digit) **M1**

Case 1, first digit 5 or 7:

$2 \times 3 \times {}^5P_3 = 2 \times 3 \times 60 = 360$ **A1**

Case 2, first digit 4 or 6: last digit from the 2 remaining evens:

$2 \times 2 \times {}^5P_3 = 240$ **A1**

Add the cases: $360 + 240$ **M1**

$= 600$ **A1**

Question 7 — 4 marks

Answer: (a) 1440; (b) 3600

(a)

$6! \times 2!$ **M1**

1440 **A1**

(b)

$7! - 1440$ **M1**

3600 **A1**

Question 8 — 5 marks

Answer: (a) 840; (b) 1946

(a)

$\binom{6}{2}\binom{8}{3}$ **M1**

840 **A1**

(b)

$\binom{14}{5} - \binom{8}{5}$ $M1$**A1**

1946 **A1**

Question 9 — 7 marks

Answer: (a) $45a^3/4$; (b) 2

(a) General term:

$\binom{5}{r}\left(\frac{ax}{2}\right)^{5-r}\left(\frac{3}{x^2}\right)^r$ **(M1)**

Power of x:

$(5-r) - 2r = -1 \Rightarrow r = 2$ **(A1)**

Substitute $r = 2$:

$\binom{5}{2}\left(\frac{a}{2}\right)^3(3)^2 = 10 \times \frac{a^3}{8} \times 9$ **(M1)**

$= \frac{90a^3}{8}$ **(A1)**

$= \frac{45a^3}{4}$ **A1**

(b) Set $\frac{45a^3}{4} = 90 \Rightarrow a^3 = 8$ **(M1)**

$a = 2$ **A1**

Question 10 — 5 marks

Answer: (a) $1 + 3x/2 - 9x^2/8$; (b) 1.01489

(a) $1 + \frac{1}{2}(3x) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}(3x)^2$ **(M1)**

$= 1 + \frac{3}{2}x - \frac{9}{8}x^2$ **A1**

(b) Set

$1 + 3x = 1.03 \Rightarrow x = 0.01$ **(M1)**

Substitute:

$1 + 1.5(0.01) - 1.125(0.01)^2 = 1.0148875$ **(M1)**

$\sqrt{1.03} \approx 1.01489$ (5 d.p.) **A1**

Every solution is also on ibmathrevision.com/hlaa/counting-and-binomials-exam-practice
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