Conditional Probability: exam practice

10 questions · 49 marks · worked solutions and markschemes

Name _____ Date _____ Score / 49

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/conditional-probability-exam-practice

1. **3 marks · Paper 1 style · no calculator**

Bag 1 holds 3 red and 2 blue counters; bag 2 holds 1 red and 4 blue. A fair coin decides which bag is used, and one counter is taken. It is red. Find the probability that it came from bag 1.

2. **4 marks · Paper 1 style · no calculator**

A committee of 4 people is to be randomly selected from a group of 5 men and 6 women. Find the exact probability that the committee contains strictly more women than men.

3. **4 marks · Paper 1 style · no calculator**

Bag 1 has 3 red and 2 blue balls; bag 2 has 1 red and 4 blue balls. A fair coin is tossed: heads, a ball is taken from bag 1; tails, from bag 2.

Given that the ball is red, find the probability that the coin showed heads. **[4]**

4. **6 marks · Paper 1 style · no calculator**

In a team of 30 judo players, 13 have won a match by throwing (T), 12 have won by hold-down (H), and 13 have won by points decision (P). 2 players have won matches by all three methods. 5 have won matches by throwing and hold-down. 4 have won matches by hold-down and points decision. 3 have won matches by throwing and points decision. Find the exact probability that a randomly selected player has won a match by throwing (T), given that they have won a match by **exactly one** method.

5. **4 marks · Paper 2 style · calculator**

Factory A makes 60% of a company's phones and factory B makes the rest. 3% of A's phones and 5% of B's phones are faulty.

A phone is found to be faulty. Find the probability that it came from factory B. **[4]**

6. **6 marks · Paper 2 style · calculator**

At a second pet shop, let the events G and T represent buying a goldfish and a tortoise respectively. The probability of buying a tortoise is twice the probability of buying a goldfish: $P(T) = 2P(G)$. It is known that G and T are **independent events**, and the probability of buying neither is 0.28. Find $P(G|T \cup G)$, the probability a customer bought a goldfish given that they bought at least one of the two pets.

7. **4 marks · Paper 2 style · calculator**

A test for a condition gives a positive result for 92% of people who have it and for 6% of people who do not. 3% of a population have the condition.

(a) Find the probability that a person chosen at random tests positive. **[2]**

(b) Given that a person tests positive, find the probability that they have the condition. **[2]**

8. **4 marks · Paper 1 style · no calculator**

In a class of 30 students, 18 study French, 12 study Spanish and 5 study both.

(a) Find the probability that a student chosen at random studies neither. **[2]**

(b) Given that a student studies French, find the probability that they also study Spanish. **[2]**

9. **5 marks · Paper 1 style · no calculator**

$P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$.

(a) Find $P(A \cap B)$. **[2]**

(b) Find $P(A \mid B)$ and state whether A and B are independent. **[3]**

10. **9 marks · Paper 2 style · calculator**

Events $A, B,$ and C are represented in a universal set. It is given that A and B are independent events, where $P(A) = 0.25$ and $P(B) = 0.52$. Events B and C are mutually exclusive. You are also given that $P(A \cap C) = 0.09$, and $P(A \cup B \cup C) = 0.95$.

1. Find $P(A \cap B)$. **[1 mark]**
2. Using the formula for the union of three sets, find $P(C)$. **[4 marks]**
3. Find $P(A' \cap B')$. **[2 marks]**
4. Find $P(A|C')$. **[2 marks]**

Answers and worked solutions

Question 1

3 marks

Answer: $3/4$

$P(R) = 0.5 \times 0.6 + 0.5 \times 0.2 = 0.4$ (M1)

$P(\text{bag 1} \mid R) = \dfrac{0.3}{0.4}$ (M1)

$= \dfrac{3}{4}$ A1

Question 2

4 marks

Answer: $23/66$

More women than men means 3 women and 1 man, or 4 women and 0 men (M1)

Count: $\binom{6}{3}\binom{5}{1} + \binom{6}{4}\binom{5}{0} = 100 + 15 = 115$ A1

Total number of committees: $\binom{11}{4} = 330$ A1

Probability: $\frac{115}{330} = \frac{23}{66}$ A1

Question 3

4 marks

Answer: $\frac{3}{4}$

$P(R) = \dfrac{1}{2} \cdot \dfrac{3}{5} + \dfrac{1}{2} \cdot \dfrac{1}{5} = \dfrac{2}{5}$ $M1$A1

$\dfrac{0.3}{0.4}$ M1

$\dfrac{3}{4}$ A1

Question 4

6 marks

Answer: $7/20$

Place 2 in $T \cap H \cap P$ and subtract it from each pairwise intersection M1

Exactly two: $T \cap H$ only $5 - 2 = 3$, $H \cap P$ only $4 - 2 = 2$, $T \cap P$ only $3 - 2 = 1$ A1

Subtract from each set total to get the single-method regions M1

T only $= 13 - 3 - 1 - 2 = 7$, H only $= 12 - 3 - 2 - 2 = 5$, P only $= 13 - 2 - 1 - 2 = 8$ A1

Exactly one method: $7 + 5 + 8 = 20$ players A1

$P(T \mid \text{exactly one}) = \frac{7}{20}$ A1

Question 5

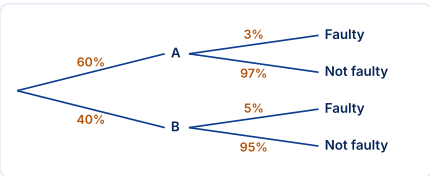
4 marks

Answer: $\frac{10}{19}$

$P(F) = 0.6(0.03) + 0.4(0.05) = 0.038$ $M1$ A1

$\dfrac{0.02}{0.038}$ M1

$\dfrac{10}{19}$ A1



Factory first, then faulty or not

Question 6

6 marks

Answer: $5/12$

Let $P(G) = g$, $P(T) = 2g$. G' and T' are also independent: $(1 - g)(1 - 2g) = 0.28$ M1

$2g^2 - 3g + 0.72 = 0$ A1

$g = 0.3$ or $g = 1.2$; reject 1.2 (a probability cannot exceed 1), so $P(G) = 0.3$ A1

$P(T \cup G) = 1 - 0.28 = 0.72$ A1

Since $G \subseteq T \cup G$:

$P(G \mid T \cup G) = \frac{P(G)}{P(T \cup G)} = \frac{0.3}{0.72}$ M1

$P(G \mid T \cup G) = \frac{5}{12} \approx 0.417$ A1

Question 7

4 marks

Answer: (a) 0.0858; (b) 0.322

(a)

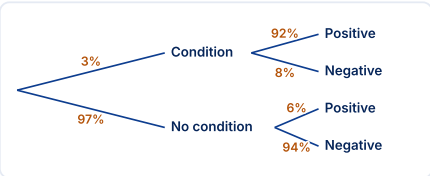
$0.03(0.92) + 0.97(0.06)$ M1

0.0858 A1

(b)

$\dfrac{0.0276}{0.0858}$ M1

0.322 A1



Condition first, then the test result

Question 8

4 marks

Answer: (a) $\frac{1}{6}$; (b) $\frac{5}{18}$

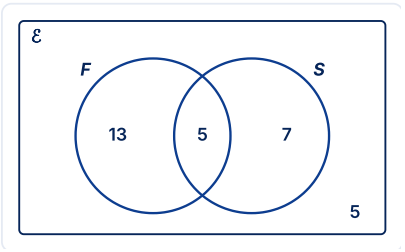
(a)

$30 - (18 + 12 - 5) = 5$ M1

$\dfrac{1}{6}$ A1

(b)

$\dfrac{5}{18}$ A2



French (F) and Spanish (S)

Question 9

5 marks

Answer: (a) $\frac{1}{5}$; (b) 0.5; independent

(a)

$0.5 + 0.4 - 0.7$ M1

0.2 A1

(b)

$\dfrac{0.2}{0.4} = 0.5$ A1

$P(A \mid B) = P(A)$ R1

independent A1

Question 10

9 marks

Answer: (a) 0.13; (b) 0.4; (c) 0.36; (d) $4/15$

Part 1:

Independent:

$P(A \cap B) = 0.25 \times 0.52 = 0.13$ A1

Part 2:

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap$ M1

B and C are mutually exclusive, so

$P(B \cap C) = P(A \cap B \cap C) = 0$ R1

$0.95 = 0.25 + 0.52 + P(C) - 0.13 - 0.09$ A1

$P(C) = 0.4$ A1

Part 3:

$P(A' \cap B') = 1 - P(A \cup B) = 1 - (0.25 + 0.52 - 0.13)$ M1

$P(A' \cap B') = 0.36$ A1

Part 4:

$P(A \cap C') = 0.25 - 0.09 = 0.16$ and $P(C') = 0.6$, so $P(A \mid C') = \frac{0.16}{0.6}$ M1

$P(A \mid C') = \frac{4}{15} \approx 0.267$ A1

Every solution is also on ibmathrevision.com/hlaa/conditional-probability-exam-practice

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