# Complex Numbers: exam practice

10 questions · 48 marks · worked solutions and markschemes

Name _____ Date _____ Score / 48

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/complex-numbers-exam-practice

**1.** **3 marks · Paper 1 style · no calculator**

Find the exact sum of the four 4th roots of the complex number $16i$.

**2.** **5 marks · Paper 2 style · calculator**

Use de Moivre's theorem to show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$.

Show this result. **[5]**

**3.** **6 marks · Paper 1 style · no calculator**

Solve the equation $z^3 = -8i$, giving your answers in the form $a + bi$.

Find the three roots. **[6]**

**4.** **3 marks · Paper 1 style · no calculator**

Find the exact modulus and the principal argument of the complex number $w = \frac{4\,\mathrm{cis}\left(\frac{5\pi}{6}\right)}{2\,\mathrm{cis}\left(\frac{\pi}{3}\right)}$.

**5.** **4 marks · Paper 1 style · no calculator**

Let $z_1 = 4\,\mathrm{cis}\,\dfrac{\pi}{3}$ and $z_2 = 2\,\mathrm{cis}\,\dfrac{\pi}{12}$.

**(a)** Find $\dfrac{z_1}{z_2}$ in modulus-argument form. **[2]**

**(b)** Hence write $\dfrac{z_1}{z_2}$ in the form $a + bi$. **[2]**

**6.** **5 marks · Paper 1 style · no calculator**

The complex number $w = 2 + 3i$ is a root of the cubic polynomial $P(z) = z^3 + pz^2 + qz + 39 = 0$, where $p$ and $q$ are real constants. Find the exact values of $p$ and $q$.

**7.** **5 marks · Paper 1 style · no calculator**

Let $w = -1 + \sqrt{3}\,i$.

**(a)** Write $w$ in modulus-argument form. **[3]**

**(b)** Hence find $w^6$. **[2]**

**8.** **6 marks · Paper 1 style · no calculator**

Let $z = \dfrac{7 + i}{1 - i}$.

**(a)** Write $z$ in the form $a + bi$, where $a, b \in \mathbb{R}$. **[3]**

**(b)** Find $|z|$ and $z^2$. **[3]**

**9.** **5 marks · Paper 1 style · no calculator**

Let $\omega = e^{\frac{2\pi i}{5}}$.

**(a)** Show that $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$. **[2]**

**(b)** Hence find the value of $\cos\dfrac{2\pi}{5} + \cos\dfrac{4\pi}{5}$. **[3]**

**10.** **6 marks · Paper 1 style · no calculator**

The cubic equation $z^3 - 5z^2 + az + b = 0$, where $a, b \in \mathbb{R}$, has a root $z = 2 + i$.

**(a)** Write down a second root. **[1]**

**(b)** Find the third root and the values of $a$ and $b$. **[5]**

# Answers and worked solutions

## Question 1

3 marks

**Answer:** $0$

The fourth roots of $16i$ are the roots of $z^4 - 16i = 0$ M1

The sum of the roots is $-\frac{\text{coefficient of } z^3}{\text{coefficient of } z^4}$, and the $z^3$ coefficient is $0$ R1

Sum of the roots $= 0$ A1

## Question 2

5 marks

**Answer:** a proof: the worked solution above is the full answer

$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$ M1

expanding: real part

$\cos^3\theta - 3\cos\theta\sin^2\theta$ $M1$A1

$= \cos^3\theta - 3\cos\theta(1 - \cos^2\theta)$ M1

$= 4\cos^3\theta - 3\cos\theta$ A1 AG

## Question 3

6 marks

**Answer:** $\sqrt{3} - i$, $2i$, $-\sqrt{3} - i$

$-8i = 8\,\text{cis}\left(-\frac{\pi}{2}\right)$ A1

$z = 2\,\text{cis}\left(\frac{-\frac{\pi}{2} + 2k\pi}{3}\right)$ M1

arguments $-\frac{\pi}{6}, \frac{\pi}{2}, \frac{7\pi}{6}$ A1

$\sqrt{3} - i$ A1

$2i$ A1

$-\sqrt{3} - i$ A1

## Question 4

3 marks

**Answer:** $(|w|)\ 2$; $(\arg w)\ \pi/2$

1. Attempt to divide moduli and subtract arguments **(M1)**
2. Modulus is $|w| = 2$ A1
3. Principal argument is $\arg(w) = \frac{5\pi}{6} - \frac{\pi}{3} = \frac{\pi}{2}$ A1

## Question 5

4 marks

**Answer:** (a) $2\,\text{cis}\frac{\pi}{4}$; (b) $\sqrt{2} + \sqrt{2}\,i$

**(a)**

$2\,\text{cis}\frac{\pi}{4}$ A1A1

**(b)**

$2\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)$ M1

$\sqrt{2} + \sqrt{2}\,i$ A1

## Question 6

5 marks

**Answer:** $(p)\ -1$; $(q)\ 1$

Coefficients are real, so the conjugate $2 - 3i$ is also a root R1

Form the quadratic factor:

$(z - (2 + 3i))(z - (2 - 3i)) = z^2 - 4z + 13$ A1

Let the third factor be $(z - \alpha)$:

compare constant terms, $-13\alpha = 39$, so $\alpha = -3$ M1

Expand

$(z^2 - 4z + 13)(z + 3) = z^3 - z^2 + z + 39$

Compare $z^2$ coefficients: $p = -1$ A1

Compare $z$ coefficients: $q = 1$ A1

## Question 7

5 marks

**Answer:** (a) $2\,\text{cis}\frac{2\pi}{3}$; (b) $64$

**(a)**

$|w| = 2$ A1

$\arg w = \frac{2\pi}{3}$ A1

$w = 2\,\text{cis}\frac{2\pi}{3}$ A1

**(b)**

$2^6\,\text{cis}(4\pi)$ M1

$64$ A1

## Question 8

6 marks

**Answer:** (a) $3 + 4i$; (b) $|z| = 5$, $z^2 = -7 + 24i$

**(a)**

$\frac{(7 + i)(1 + i)}{(1 - i)(1 + i)}$ M1

$= \frac{6 + 8i}{2}$ A1

$= 3 + 4i$ A1

**(b)**

$|z| = 5$ A1

$z^2 = 9 + 24i - 16$ M1

$= -7 + 24i$ A1

## Question 9

5 marks

**Answer:** (b) $-\frac{1}{2}$

**(a)**

geometric series: $\frac{1 - \omega^5}{1 - \omega}$ M1

$\omega^5 = 1$, $\omega \neq 1$, so the sum is $0$ A1 AG

**(b)**

real parts:

$1 + 2\cos\frac{2\pi}{5} + 2\cos\frac{4\pi}{5} = 0$ $M1$ A1

$-\frac{1}{2}$ A1

## Question 10

6 marks

**Answer:** (a) $2 - i$; (b) $z = 1$, $a = 9$, $b = -5$

**(a)**

$2 - i$ A1

**(b)**

sum of roots $= 5$ M1

third root $= 1$ A1

$(z^2 - 4z + 5)(z - 1) = z^3 - 5z^2 + 9z - 5$ M1

$a = 9$ A1

$b = -5$ A1

Every solution is also on ibmathrevision.com/hlaa/complex-numbers-exam-practice
Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.