IB MATHS AA HL · UNIT 5

Calculus: exam practice

10 questions · 45 marks · worked solutions and markschemes

Name _____ Date _____ Score / 45

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/calculus-exam-practice

1. **4 marks · Paper 1 style · no calculator**

Evaluate the limit $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$.

2. **4 marks · Paper 1 style · no calculator**

Differentiate the function $f(x) = e^{3x^2 - 5}$ with respect to x.

3. **3 marks · Paper 1 style · no calculator**

Find $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 - x - 6}$.

Find the limit. **[3]**

4. **3 marks · Paper 1 style · no calculator**

Find the Maclaurin series for $\cos(2x)$ up to and including the term in x^4.

Find the series. **[3]**

5. **4 marks · Paper 2 style · calculator**

A spherical balloon is inflated at a rate of $50\ \text{cm}^3\,\text{s}^{-1}$.

Find the rate at which the radius increases when the radius is 10 cm. **[4]**

6. **4 marks · Paper 2 style · calculator**

A particle moves with velocity $v(t) = 6t - t^2$ m s^{-1} for $0 \leq t \leq 8$.

Find the total distance travelled. **[4]**

7. **6 marks · Paper 2 style · calculator**

Find the gradient of the tangent to the curve $x^2 + y^2 - 3xy = 5$ at the point $(1, -1)$.

8. **5 marks · Paper 1 style · no calculator**

Solve the differential equation $\frac{dy}{dx} = xy^2$, given that $y = 1$ when $x = 0$.

Find y in terms of x. **[5]**

9. **8 marks · Paper 1 style · no calculator**

Find the general solution of the differential equation $\frac{dy}{dx} + \frac{3}{x}y = x$ for $x > 0$.

10. **4 marks · Paper 1 style · no calculator**

Find $\int \frac{2x}{x^2+5}\,dx$ and hence evaluate $\int_0^2 \frac{2x}{x^2+5}\,dx$ exactly.

(a) The indefinite integral. **[2]**

(b) The definite integral. **[2]**

Answers and worked solutions

Question 1 — 4 marks

Answer: 4

Direct substitution gives $\frac{0}{0}$, so simplify first.

Factorise the numerator:

$x^2 - 4 = (x-2)(x+2)$ $M1$ A1

Cancel $(x-2)$ (valid since $x \neq 2$): $\frac{x^2-4}{x-2} = x + 2$ M1

Evaluate: $\lim_{x \to 2}(x+2) = 4$ A1

Question 2 — 4 marks

Answer:

$f'(x) = 6xe^(3x^2 - 5)$

Apply the chain rule with $u = 3x^2 - 5$: $f'(x) = e^u \cdot \frac{du}{dx}$ M1

Inner derivative: $\frac{du}{dx} = 6x$ A1

Derivative of the outer function: e^{3x^2-5} A1

Simplify: $f'(x) = 6xe^{3x^2-5}$ A1

Question 3 — 3 marks

Answer: $\frac{6}{5}$

$\frac{(x-3)(x+3)}{(x-3)(x+2)}$ $M1$A1

$\frac{6}{5}$ A1

Question 4 — 3 marks

Answer: $1 - 2x^2 + \frac{2}{3}x^4$

$\cos u = 1 - \frac{u^2}{2} + \frac{u^4}{24}$ M1

$u = 2x$ M1

$1 - 2x^2 + \frac{2}{3}x^4$ A1

Question 5 — 4 marks

Answer: $0.0398\ \text{cm s}^{-1}$

$\frac{dV}{dr} = 4\pi r^2$ A1

$\frac{dr}{dt} = \frac{50}{4\pi(100)}$ $M1$A1

$0.0398\ \text{cm s}^{-1}$ A1

Question 6 — 4 marks

Answer: $\frac{152}{3}\ m$

$v = 0$ at $t = 6$ A1

$\int_0^6 v\,dt + \left|\int_6^8 v\,dt\right|$ M1

$36 + \frac{44}{3}$ A1

$\frac{152}{3}\ m$ A1

Question 7 — 6 marks

Answer: 1

Differentiate implicitly (product rule on $3xy$):

$2x + 2y\frac{dy}{dx} - 3\left(y + x\frac{dy}{dx}\right) = 0$

$M1A1$A1

Substitute $x = 1$, $y = -1$:

$2 - 2\frac{dy}{dx} + 3 - 3\frac{dy}{dx} = 0$ M1

$5\frac{dy}{dx} = 5$ A1

Gradient of the tangent: $\frac{dy}{dx} = 1$ A1

Check: $1 + 1 - 3(1)(-1) = 5$, so $(1, -1)$ lies on the curve.

Question 8 — 5 marks

Answer: $y = \frac{2}{2 - x^2}$

$\int y^{-2}\,dy = \int x\,dx$ M1

$-\frac{1}{y} = \frac{x^2}{2} + c$ A1A1

$c = -1$ A1

$y = \frac{2}{2 - x^2}$ A1

Question 9 — 8 marks

Answer: $y = x^2/5 + C/x^3$

Find the integrating factor:

$I(x) = e^{\int (3/x)\,dx}$ M1

$= e^{3\ln x} = x^3$ A1

Multiply the DE by x^3:

$x^3\frac{dy}{dx} + 3x^2 y = x^4$ M1

Recognise the LHS as a product derivative: $\frac{d}{dx}(x^3 y) = x^4$ A1

Integrate:

$x^3 y = \int x^4\,dx = \frac{x^5}{5} + C$ M1 A1 A1

Isolate y: $y = \frac{x^2}{5} + Cx^{-3}$ A1

Question 10 — 4 marks

Answer: (a) $\ln(x^2 + 5) + c$; (b) $\ln\frac{9}{5}$

(a)

$\ln(x^2 + 5) + c$ A2

(b)

$\ln 9 - \ln 5$ M1

$\ln\frac{9}{5}$ A1

Every solution is also on ibmathrevision.com/hlaa/calculus-exam-practice

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