Binomial Theorem: exam practice

10 questions · 52 marks · worked solutions and markschemes

Name _____ Date _____ Score / 52

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/binomial-theorem-exam-practice

1. **4 marks · Paper 1 style · no calculator**

The coefficient of x^3 in the expansion of $(k+2x)^6$ is -1280.

Find the value of k. **[4]**

2. **5 marks · Paper 1 style · no calculator**

Consider the expansion of $\left(3x^2-\frac{2}{x}\right)^9$.

Find the term independent of x. **[5]**

3. **5 marks · Paper 1 style · no calculator**

Find the coefficient of x^4 in the expansion of $(2-x)(1+2x)^7$.

Find this coefficient. **[5]**

4. **4 marks · Paper 2 style · calculator**

The fourth term in the expansion of $\left(px+\frac{1}{x}\right)^8$, in descending powers of x, is $\frac{5}{2}x^2$.

Find the value of p, given that $p>0$. **[4]**

5. **6 marks · Paper 1 style · no calculator**

In the expansion of $(1+ax)^n$, where $n\in\mathbb{Z}^+$, the coefficient of x is 15 and the coefficient of x^2 is 90.

Find the value of n and the value of a. **[6]**

6. **5 marks · Paper 1 style · no calculator**

Let $f(x) = \dfrac{1}{\sqrt{1-4x}}$.

(a) Find the first three terms in the expansion of $f(x)$ in ascending powers of x. **[4]**

(b) State the values of x for which the expansion is valid. **[1]**

7. **5 marks · Paper 2 style · calculator**

Consider the function $f(x) = \dfrac{2}{2+x}$.

(a) Find the first four terms in the binomial expansion of $f(x)$ in ascending powers of x. **[4]**

(b) Find the range of values of x for which the expansion is valid. **[1]**

8. **7 marks · Paper 2 style · calculator**

Let $g(x) = (8+3x)^{\frac{1}{3}}$.

(a) Show that $g(x) = 2\left(1+\dfrac{3x}{8}\right)^{\frac{1}{3}}$. **[1]**

(b) Find the expansion of $g(x)$ up to and including the term in x^2. **[4]**

(c) Use your answer with $x = 0.08$ to estimate $\sqrt[3]{8.24}$ to 6 significant figures. **[2]**

9. **5 marks · Paper 1 style · no calculator**

Let $h(x) = \dfrac{1+x}{(1-2x)^2}$.

(a) Expand $(1-2x)^{-2}$ in ascending powers of x up to and including the term in x^3. **[3]**

(b) Hence find the coefficient of x^3 in the expansion of $h(x)$. **[2]**

10. **6 marks · Paper 1 style · no calculator**

The expansion of $(1+ax)^{-3}$ in ascending powers of x has x-coefficient -6.

(a) Find the value of a. **[2]**

(b) Find the coefficient of x^3. **[3]**

(c) State the values of x for which the expansion converges. **[1]**

Answers and worked solutions

Question 1

4 marks

Answer: -2

$\binom{6}{3}k^3(2)^3 = 160k^3$ $M1A1$

$160k^3 = -1280$ M1

$k = -2$ A1

Question 2

5 marks

Answer: 145152

$T_{r+1} = \binom{9}{r}(3x^2)^{9-r}\left(-\frac{2}{x}\right)^r$

(M1)

power of x: $18 - 2r - r = 0$ M1

$r = 6$ A1

$\binom{9}{6}3^3(-2)^6 = 84 \times 27 \times 64$

(M1)

145152 A1

Question 3

5 marks

Answer: 840

$(1+2x)^7$: x^4 term

$\binom{7}{4}2^4x^4 = 560x^4$, x^3 term

$\binom{7}{3}2^3x^3 = 280x^3$ $M1A1A1$

$2(560) - 1(280)$ M1

840 A1

Question 4

4 marks

Answer: $p = 0.537$

fourth term

$= \binom{8}{3}(px)^5\left(\frac{1}{x}\right)^3 = 56p^5x^2$

$M1A1$

$56p^5 = \frac{5}{2}$ M1

$p = 0.537$ A1

Question 5

6 marks

Answer: $n = 5$, $a = 3$

$na = 15$ A1

$\frac{n(n-1)}{2}a^2 = 90$ A1

$\frac{n(n-1)}{2} \cdot \frac{225}{n^2} = 90$ M1

$225(n-1) = 180n$ A1

$n = 5$ A1

$a = 3$ A1

Question 6

5 marks

Answer: (a) $1 + 2x + 6x^2$; (b) $|x| < \frac{1}{4}$

(a)

$(1-4x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-4x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-4x)^2$

$M1A1$

$= 1 + 2x + 6x^2$ A1A1

(b)

$|x| < \frac{1}{4}$ A1

Question 7

5 marks

Answer: (a) $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8}$; (b) $-2 < x < 2$

(a)

$\left(1 + \frac{x}{2}\right)^{-1}$ M1

$= 1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8}$ A1A1A1

(b)

$-2 < x < 2$ A1

Question 8

7 marks

Answer: (b) $2 + \frac{x}{4} - \frac{x^2}{32}$; (c) 2.01980

(a)

$8^{\frac{1}{3}}\left(1 + \frac{3x}{8}\right)^{\frac{1}{3}} = 2\left(1 + \frac{3x}{8}\right)^{\frac{1}{3}}$

A1 AG

(b)

$2\left[1 + \frac{1}{3} \cdot \frac{3x}{8} + \frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}\left(\frac{3x}{8}\right)^2\right]$

$M1A1$

$= 2 + \frac{x}{4} - \frac{x^2}{32}$ A1A1

(c)

$2 + 0.02 - 0.0002$ M1

2.01980 A1

Question 9

5 marks

Answer: (a) $1 + 4x + 12x^2 + 32x^3$; (b) 44

(a)

$1 + (-2)(-2x) + \frac{(-2)(-3)}{2!}(-2x)^2 + \frac{(-2)(-3)(-4)}{3!}(-$

M1

$= 1 + 4x + 12x^2 + 32x^3$ A2

(b)

$32 + 12$ M1

44 A1

Question 10

6 marks

Answer: (a) 2; (b) -80; (c) $|x| < \frac{1}{2}$

(a)

$-3a = -6$ M1

$a = 2$ A1

(b)

$\frac{(-3)(-4)(-5)}{3!}(2x)^3$ $M1A1$

-80 A1

(c)

$|x| < \frac{1}{2}$ A1

Every solution is also on ibmathrevision.com/hlaa/binomial-theorem-exam-practice

Our own original questions, written and checked by IB Math Revision. IB Math Revision is independent and is not affiliated with, endorsed by or connected to the International Baccalaureate Organization. This work/product/service has been developed independently from and is not endorsed by the International Baccalaureate Organization. International Baccalaureate, Baccalauréat International, Bachillerato Internacional and IB are registered trademarks owned by the International Baccalaureate Organization.