Applications of Differentiation: exam practice

10 questions · 47 marks · worked solutions and markschemes

Name _____ Date _____ Score / 47

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/applications-of-differentiation-exam-practice

1. **4 marks · Paper 2 style · calculator**

The profit from selling x hundred items is $P(x) = -2x^3 + 27x^2 - 84x - 40$ thousand dollars, for $0 \leq x \leq 10$.

Find the number of items that maximises profit and the maximum profit. **[4]**

2. **4 marks · Paper 2 style · calculator**

Let $f(x) = \dfrac{x^2}{x-1}$, $x \neq 1$.

Find the coordinates of the stationary points. **[4]**

3. **4 marks · Paper 2 style · calculator**

A wire of length 40 cm is bent into a rectangle.

Show that the largest area is when the rectangle is a square, and find this area. **[4]**

4. **4 marks · Paper 1 style · no calculator**

Let $f(x) = e^x - 2x$.

Find the exact coordinates of the stationary point and show that it is a minimum. **[4]**

5. **5 marks · Paper 2 style · calculator**

An open box with a square base has volume $32\ \text{m}^3$. The base has side x m.

Find the dimensions that minimise the surface area. **[5]**

6. **6 marks · Paper 1 style · no calculator**

Let $f(x) = x^3 - 6x^2 + 9x + 1$.

Find the coordinates of the stationary points and classify them. **[6]**

7. **4 marks · Paper 1 style · no calculator**

Let $f(x) = x \ln x$ for $x > 0$.

Find the coordinates of the minimum point of f. **[4]**

8. **5 marks · Paper 2 style · calculator**

A rectangle is drawn under the curve $y = 12 - x^2$ with two vertices on the x-axis and two on the curve, symmetric about the y-axis.

Find the maximum area of the rectangle. **[5]**

9. **5 marks · Paper 2 style · calculator**

A point P lies on the curve $y = \sqrt{x}$. Find the coordinates of P closest to $A(4, 0)$.

Find P. **[5]**

10. **6 marks · Paper 2 style · calculator**

A closed cylinder has volume $500\ \text{cm}^3$.

(a) Show that its surface area is $S = 2\pi r^2 + \dfrac{1000}{r}$. **[2]**

(b) Find the minimum surface area. **[4]**

Answers and worked solutions

Question 1 — 4 marks

Answer: 700 items, $\$9000$

$P'(x) = -6x^2 + 54x - 84 = 0$ M1

$x = 2$ or $x = 7$ A1

maximum at $x = 7$: 700 items A1

$P(7) = 9$, so $\$9000$ A1

Question 2 — 4 marks

Answer: $(0, 0)$ and $(2, 4)$

quotient rule:

$f'(x) = \dfrac{x^2 - 2x}{(x-1)^2}$ $M1A1$

$(0, 0)$ A1

$(2, 4)$ A1

Question 3 — 4 marks

Answer: $100\ \text{cm}^2$

$A = x(20 - x)$ $M1A1$

$20 - 2x = 0$, $x = 10$ (a square) A1

$100\ \text{cm}^2$ A1

Question 4 — 4 marks

Answer: $(\ln 2,\ 2 - 2\ln 2)$

$f'(x) = e^x - 2 = 0$ M1

$x = \ln 2$ A1

$(\ln 2,\ 2 - 2\ln 2)$ A1

$f''(x) = e^x > 0$, minimum R1

Question 5 — 5 marks

Answer: base $4\ m$ by $4\ m$, height $2\ m$

$h = \dfrac{32}{x^2}$ A1

$S = x^2 + \dfrac{128}{x}$ A1

$2x - \dfrac{128}{x^2} = 0$ M1

$x = 4$ A1

$h = 2$ A1

Question 6 — 6 marks

Answer: maximum $(1, 5)$, minimum $(3, 1)$

$f'(x) = 3x^2 - 12x + 9$ A1

$3(x-1)(x-3) = 0$ M1

$(1, 5)$ A1

$(3, 1)$ A1

$f''(1) = -6 < 0$: maximum;
$f''(3) = 6 > 0$: minimum $M1$ A1

Question 7 — 4 marks

Answer: $\left(\dfrac{1}{e},\ -\dfrac{1}{e}\right)$

$f'(x) = \ln x + 1$ A1

$\ln x = -1$ M1

$x = e^{-1}$ A1

$\left(\dfrac{1}{e},\ -\dfrac{1}{e}\right)$ A1

Question 8 — 5 marks

Answer: 32

$A = 2x(12 - x^2)$ $M1A1$

$\dfrac{dA}{dx} = 24 - 6x^2 = 0$ M1

$x = 2$ A1

$A = 32$ A1

Question 9 — 5 marks

Answer: $\left(\dfrac{7}{2},\ \sqrt{3.5}\right)$

$D^2 = (x-4)^2 + x$ $M1A1$

$\dfrac{d(D^2)}{dx} = 2(x-4) + 1 = 0$ M1

$x = \dfrac{7}{2}$ A1

$P\left(\dfrac{7}{2},\ \sqrt{\dfrac{7}{2}}\right)$ A1

Question 10 — 6 marks

Answer: (b) $349\ \text{cm}^2$

(a)

$h = \dfrac{500}{\pi r^2}$ M1

$S = 2\pi r^2 + 2\pi r \cdot \dfrac{500}{\pi r^2}$ A1 AG

(b)

$\dfrac{dS}{dr} = 4\pi r - \dfrac{1000}{r^2} = 0$ $M1$ A1

$r = 4.30$ A1

$S = 349\ \text{cm}^2$ A1

Every solution is also on ibmathrevision.com/hlaa/applications-of-differentiation-exam-practice
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