Advanced Random Variables: exam practice

10 questions · 42 marks · worked solutions and markschemes

Name _____ Date _____ Score / 42

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/advanced-random-variables-exam-practice

1. **2 marks · Paper 2 style · calculator**

A continuous random variable T follows a uniform distribution with probability density function $f(t) = 0.5$ for $2 \leq t \leq 4$. Find $P(T \geq 3.5)$.

2. **3 marks · Paper 2 style · calculator**

A continuous random variable X has a probability density function given by $f(x) = k(3 - x)$ for $0 \leq x \leq 3$, and $f(x) = 0$ otherwise. Find the exact value of the constant k.

3. **4 marks · Paper 1 style · no calculator**

A continuous random variable Θ has the probability density function $f(\theta) = \cos\theta$ for $0 \leq \theta \leq \frac{\pi}{2}$. Find the exact probability that $\Theta < \frac{\pi}{6}$.

4. **5 marks · Paper 1 style · no calculator**

For the same continuous random variable X with PDF $f(x) = \frac{3}{8}x^2$ $(0 \leq x \leq 2)$, find the exact variance of X.

5. **4 marks · Paper 2 style · calculator**

The time a student spends on homework each night has mean 75 minutes and standard deviation 20 minutes. The distribution is not known.

Using the central limit theorem, estimate the probability that the mean time over 40 nights is less than 70 minutes. **[4]**

6. **3 marks · Paper 1 style · no calculator**

A discrete random variable X has an expected value $\mathrm{E}(X) = 3$ and a variance $\mathrm{Var}(X) = 4$. Find the exact values of $\mathrm{E}(5X - 2)$ and $\mathrm{Var}(5X - 2)$.

7. **4 marks · Paper 2 style · calculator**

Let S be the score on a biased die where $\mathbf{E}(S) = 4.2$ and $\mathbf{Var}(S) = 1.5$. A game pays out a prize, Y euros, based on the formula $Y = 3S - 5$. Find the expected payout and the standard deviation of the payout. Give your answers correct to 3 significant figures.

8. **4 marks · Paper 2 style · calculator**

The masses of apples are normally distributed with mean 160 g and standard deviation 12 g. A bag holds 9 apples chosen at random.

(a) Write down the distribution of the mean mass $\bar{X}$ of the 9 apples. **[2]**

(b) Find $P(\bar{X} > 165)$. **[2]**

9. **6 marks · Paper 2 style · calculator**

A continuous random variable X has the probability density function $f(x) = \frac{3}{8}x^2$ for $0 \leq x \leq 2$. Calculate the expected value $\mathbf{E}(X)$ and the variance $\mathbf{Var}(X)$.

10. **7 marks · Paper 1 style · no calculator**

A continuous random variable X has probability density function $f(x) = kx(4 - x)$ for $0 \leq x \leq 4$, and 0 otherwise.

(a) Find k. **[3]**

(b) Find $\mathrm{Var}(X)$. **[4]**

Answers and worked solutions

Question 1

2 marks

Answer: 0.25

Integrate the pdf from 3.5 to 4:

$P(T \geq 3.5) = \int_{3.5}^{4} 0.5\,dt = 0.5 \times 0.5$ M1

$P(T \geq 3.5) = 0.25$ A1

Question 2

3 marks

Answer: $2/9$

Total probability is 1:

$\int_0^3 k(3-x)\,dx = 1$ M1

Integrate:

$k\left[3x - \frac{x^2}{2}\right]_0^3 = k\left(9 - \frac{9}{2}\right) = \frac{9k}{2}$ A1

Solve $\frac{9k}{2} = 1$: $k = \frac{2}{9}$ A1

Question 3

4 marks

Answer: $1/2$

Integrate the pdf:

$P\left(\Theta < \frac{\pi}{6}\right) = \int_0^{\pi/6} \cos\theta\,d\theta$ M1

Antiderivative: $= \left[\sin\theta\right]_0^{\pi/6}$ A1

Substitute limits: $= \sin\frac{\pi}{6} - \sin 0$ A1

$= \frac{1}{2}$ A1

Question 4

5 marks

Answer: $3/20$

Set up

$E(X) = \int_0^2 x\left(\frac{3}{8}x^2\right)dx = \left[\frac{3x^4}{32}\right]_0^2$ M1

$E(X) = \frac{3}{2}$ A1

Set up

$E(X^2) = \int_0^2 \frac{3}{8}x^4\,dx = \left[\frac{3x^5}{40}\right]_0^2$ M1

$E(X^2) = \frac{96}{40} = \frac{12}{5}$ A1

Use

$\mathrm{Var}(X) = E(X^2) - [E(X)]^2 = \frac{12}{5} - \frac{9}{4} = \frac{3}{20}$ A1

Question 5

4 marks

Answer: 0.0569

$n = 40$ is large, so

$\bar{T} \approx N\left(75,\ \frac{400}{40}\right)$ R1A1

$P(\bar{T} < 70)$ M1

0.0569 A1

Question 6

3 marks

Answer: ($E(5X-2)$ 13; ($\mathrm{Var}(5X-2)$ 100

Use $\mathrm{E}(aX+b) = a\mathrm{E}(X) + b$:

$\mathrm{E}(5X-2) = 5(3) - 2 = 13$ A1

Use $\mathrm{Var}(aX+b) = a^2\mathrm{Var}(X)$:

$\mathrm{Var}(5X-2) = 5^2 \times 4$ M1

$\mathrm{Var}(5X-2) = 100$ A1

Question 7

4 marks

Answer: (Expected payout (€) 7.6; (Standard deviation (€) 3.67

Expected payout:

$\mathrm{E}(Y) = 3\mathrm{E}(S) - 5 = 3(4.2) - 5 = 7.6$ euros A1

Use $\mathrm{Var}(aS+b) = a^2\mathrm{Var}(S)$:

$\mathrm{Var}(Y) = 3^2 \times 1.5$ M1

$\mathrm{Var}(Y) = 13.5$ A1

Standard deviation:

$\sigma_Y = \sqrt{13.5} = 3.67$ euros (3 s.f.) A1

Question 8

4 marks

Answer: (a) $\bar{X} \sim N(160,\ 4^2)$; (b) 0.106

(a)

$\bar{X} \sim N\left(160,\ \frac{12^2}{9}\right)$ A1A1

(b)

$N(160, 4^2)$ M1

0.106 A1

Question 9

6 marks

Answer: ($E(X)$ 1.5; ($\mathrm{Var}(X)$ 0.15

Use $\mathrm{E}(X) = \int x f(x)\,dx$:

$\mathrm{E}(X) = \int_0^2 \frac{3}{8}x^3\,dx$ M1

$\mathrm{E}(X) = \frac{3}{8}\left[\frac{x^4}{4}\right]_0^2 = \frac{3}{8} \times 4 = 1.5$ A1

Find $\mathrm{E}(X^2) = \int_0^2 \frac{3}{8}x^4\,dx$ M1

$\mathrm{E}(X^2) = \frac{3}{8}\left[\frac{x^5}{5}\right]_0^2 = \frac{3}{8} \times \frac{32}{5} = 2.4$ A1

Use $\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$:

$\mathrm{Var}(X) = 2.4 - 1.5^2$ M1

$\mathrm{Var}(X) = 0.15$ A1

Question 10

7 marks

Answer: (a) $\frac{3}{32}$; (b) $\frac{4}{5}$

(a)

$\int_0^4 kx(4-x)\,dx = 1$ M1

$k \cdot \frac{32}{3} = 1$ A1

$k = \frac{3}{32}$ A1

(b)

$E(X) = 2$ by symmetry A1

$E(X^2) = \int_0^4 \frac{3}{32}x^3(4-x)\,dx = \frac{24}{5}$ M1A1

$\mathrm{Var}(X) = \frac{24}{5} - 4 = \frac{4}{5}$ A1

Every solution is also on ibmathrevision.com/hlaa/advanced-random-variables-exam-practice
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