# Advanced Integration: exam practice

10 questions · 47 marks · worked solutions and markschemes

Name _____ Date _____ Score / 47

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/advanced-integration-exam-practice

**1.** **3 marks · Paper 1 style · no calculator**

Find the indefinite integral:

$$\int 5^x \, dx$$

**2.** **4 marks · Paper 1 style · no calculator**

Use the method of integration by parts to find:

$$\int xe^{3x} \, dx$$

**3.** **3 marks · Paper 2 style · calculator**

The region bounded by $y = \sqrt{x}$, the $x$-axis and $x = 4$ is rotated $2\pi$ about the $x$-axis.

Find the exact volume of the solid formed. **[3]**

**4.** **4 marks · Paper 1 style · no calculator**

Use the substitution $u = 1 + \sin x$ to find $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)^2}\,dx$.

Find the integral. **[4]**

**5.** **5 marks · Paper 1 style · no calculator**

The region bounded by the curve $y = e^{2x}$, the $y$-axis, the $x$-axis, and the vertical line $x = \ln 2$ is rotated $360°$ about the $x$-axis. Find the exact volume of the solid generated.

**6.** **5 marks · Paper 1 style · no calculator**

Use the substitution $u = x^2 + 1$ to evaluate exactly the definite integral $\int_0^1 x\sqrt{x^2+1}\,dx$.

**7.** **5 marks · Paper 1 style · no calculator**

Find $\int x^2 e^x \, dx$.

Find the integral. **[5]**

**8.** **5 marks · Paper 2 style · calculator**

The region bounded by $y = x^2$ and $y = 2x$ is rotated $2\pi$ about the $y$-axis.

Find the exact volume of the solid formed. **[5]**

**9.** **6 marks · Paper 1 style · no calculator**

Find $\int e^x \sin x \, dx$.

Find the integral. **[6]**

**10.** **7 marks · Paper 1 style · no calculator**

Use **repeated** integration by parts to evaluate the definite integral exactly:

$$\int_0^1 x^2 e^{-x} \, dx$$

# Answers and worked solutions

## Question 1

3 marks

**Answer:** $5^x/\ln(5)+C$

Write $5^x = e^{x\ln 5}$ (or use $\int a^x\,dx = \frac{a^x}{\ln a}+C$): M1

$\int 5^x\,dx = \frac{5^x}{\ln 5}$ A1

$+C$ A1

## Question 2

4 marks

**Answer:**

$xe^{(}3x)/3 - e^{(}3x)/9 + C$

Let $u = x$ and $\frac{dv}{dx} = e^{3x}$, so $\frac{du}{dx} = 1$ and $v = \frac{1}{3}e^{3x}$ $M1A1$

Apply $\int u\,\frac{dv}{dx}\,dx = uv - \int v\,\frac{du}{dx}\,dx$: $\frac{x}{3}e^{3x} - \int \frac{1}{3}e^{3x}\,dx$ A1

$= \frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C$ A1

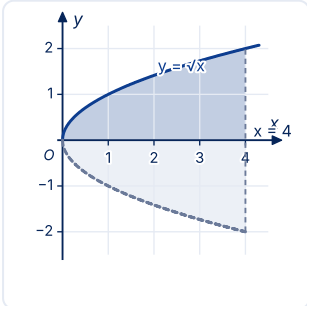
## Question 3

3 marks

**Answer:** $8\pi$

$\pi\int_0^4 x\,dx$ $M1A1$

$8\pi$ A1




Volume $= \pi\int_0^4 x\,\mathrm{d}x = 8\pi$

## Question 4

4 marks

**Answer:** $\frac{1}{2}$

$du = \cos x\,dx$ A1

limits $1$ to $2$ A1

$\int_1^2 u^{-2}\,du = \left[-u^{-1}\right]_1^2$ M1

$\frac{1}{2}$ A1

## Question 5

5 marks

**Answer:** $15\pi/4$

Volume of revolution:

$V = \pi\int_0^{\ln 2}\left(e^{2x}\right)^2 dx$ M1

$V = \pi\int_0^{\ln 2} e^{4x}\,dx$ A1

Integrate: $V = \pi\left[\frac{1}{4}e^{4x}\right]_0^{\ln 2}$ M1

Substitute the limits:

$\frac{\pi}{4}\left(e^{4\ln 2} - e^0\right) = \frac{\pi}{4}\left(2^4 - 1\right)$ M1

$V = \frac{15\pi}{4}$ A1

## Question 6

5 marks

**Answer:** $(2\sqrt{2}-1)/3$

Differentiate the substitution:

$du = 2x\,dx \implies x\,dx = \frac{1}{2}du$ M1

Change the limits: when $x = 0$, $u = 1$; when $x = 1$, $u = 2$ M1

Transform the integral: $\int_1^2 \frac{1}{2}u^{1/2}\,du$ A1

Integrate:

$\left[\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\right]_1^2 = \left[\frac{1}{3}u^{3/2}\right]_1^2$ A1

Evaluate: $\frac{1}{3}\left(2^{3/2}-1\right) = \frac{2\sqrt{2}-1}{3}$ A1

## Question 7

5 marks

**Answer:** $e^x(x^2-2x+2)+c$

parts: $u = x^2$, $dv = e^x\,dx$ M1

$x^2e^x - \int 2xe^x\,dx$ A1

parts again M1

$x^2e^x - 2xe^x + 2e^x$ A1

$+c$ A1

## Question 8

5 marks

**Answer:** $\frac{8\pi}{3}$

meet at $(0,0)$ and $(2,4)$ A1

$x^2 = y$ and $x^2 = \frac{y^2}{4}$ A1

$\pi\int_0^4 \left(y - \frac{y^2}{4}\right)dy$ $M1A1$

$\frac{8\pi}{3}$ A1

## Question 9

6 marks

**Answer:** $\frac{e^x}{2}(\sin x - \cos x) + c$

parts once:

$-e^x\cos x + \int e^x\cos x\,dx$ $M1A1$

parts again:

$e^x\sin x - \int e^x\sin x\,dx$ $M1A1$

$2I = e^x(\sin x - \cos x)$ M1

$I = \frac{e^x}{2}(\sin x - \cos x) + c$ A1

## Question 10

7 marks

**Answer:** $2 - 5/e$

First application: $u = x^2$, $\frac{dv}{dx} = e^{-x}$, so $\frac{du}{dx} = 2x$, $v = -e^{-x}$ M1

$\int_0^1 x^2e^{-x}\,dx = \left[-x^2e^{-x}\right]_0^1 + \int_0^1 2xe^{-x}\,dx$ A1

Second application: $u = 2x$, $\frac{dv}{dx} = e^{-x}$, so $\frac{du}{dx} = 2$, $v = -e^{-x}$ M1

$\int_0^1 2xe^{-x}\,dx = [-2xe^{-x}]_0^1 + \int_0^1 2e^{-x}\,dx = [-2xe^{-x} - 2e^{-x}]$ A1

Combine: $\left[-e^{-x}(x^2+2x+2)\right]_0^1$ A1

Substitute the limits: $-5e^{-1} - (-2)$ (M1)

$= 2 - \frac{5}{e}$ A1


Every solution is also on ibmathrevision.com/hlaa/advanced-integration-exam-practice
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