IB MATHS AA HL · UNIT 5

Advanced Differentiation: exam practice

10 questions · 47 marks · worked solutions and markschemes

Name _____ Date _____ Score / 47

- Answer every question before you look at the solutions. They start after the last question, with the marks for each line: M for method, A for accuracy, R for reasoning.
- Online, with more practice: ibmathrevision.com/hlaa/advanced-differentiation-exam-practice

1. **2 marks · Paper 1 style · no calculator**

Find the exact derivative of the function $f(x) = 5^x$ with respect to x.

2. **3 marks · Paper 1 style · no calculator**

Given the function $y = \arctan(3x)$, find an expression for $\frac{dy}{dx}$.

3. **4 marks · Paper 1 style · no calculator**

Find $\dfrac{dy}{dx}$ for the curve $e^y + x^2y = 4$ at the point $(2, 0)$.

Find the gradient. **[4]**

4. **5 marks · Paper 1 style · no calculator**

Find the exact gradient of the tangent to the curve $y = \arccos(x^2)$ at the point where $x = \frac{1}{\sqrt{2}}$.

5. **5 marks · Paper 1 style · no calculator**

Use L'Hôpital's rule to evaluate exactly the limit $\lim_{x \to 0} \frac{e^{2x} - 1 - 2x}{x^2}$.

6. **5 marks · Paper 2 style · calculator**

A spherical balloon is being inflated such that its volume increases at a constant rate of $15\ \text{cm}^3\ \text{s}^{-1}$. Calculate the rate of increase of its radius at the instant when the volume of the balloon is $288\pi\ \text{cm}^3$.

7. **5 marks · Paper 1 style · no calculator**

Find the equation of the tangent to the curve $y^3 + xy = 10$ at the point $(9, 1)$.

Find the equation. **[5]**

8. **5 marks · Paper 2 style · calculator**

The curve $x^2 - xy + y^2 = 12$ has two points where the tangent is horizontal.

Find the coordinates of these points. **[5]**

9. **7 marks · Paper 2 style · calculator**

A tracking camera is situated $2\ \text{km}$ away from a launch pad. A rocket is launched vertically upwards. When the rocket is exactly $3\ \text{km}$ above the ground, its vertical velocity is $0.4\ \text{km s}^{-1}$. Calculate the exact rate of change of the camera's angle of elevation (θ) at this instant.

10. **6 marks · Paper 1 style · no calculator**

A curve has equation $x^2 + 3xy + y^2 = 11$.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y. **[4]**

(b) Find the gradient of the curve at $(1, 2)$. **[2]**

Answers and worked solutions

Question 1 — 2 marks

Answer: $f'(x) = 5^x \ln(5)$

Use the standard result $\frac{d}{dx}(a^x) = a^x \ln a$ with $a = 5$: **(M1)**

$f'(x) = 5^x \ln 5$ **A1**

Question 2 — 3 marks

Answer: $dy/dx = 3/(1 + 9x^2)$

Use $\frac{d}{dx}(\arctan u) = \frac{1}{1+u^2}\frac{du}{dx}$ with $u = 3x$ (chain rule): **M1**

$\frac{du}{dx} = 3$ **A1**

$\frac{dy}{dx} = \frac{3}{1+9x^2}$ **A1**

Question 3 — 4 marks

Answer: 0

$e^y \frac{dy}{dx} + 2xy + x^2\frac{dy}{dx} = 0$ $M1$ **A1**

at $(2, 0)$: $5\frac{dy}{dx} + 0 = 0$ **A1**

$\frac{dy}{dx} = 0$ **A1**

Question 4 — 5 marks

Answer: $-2\sqrt{6}/3$

Use $\frac{d}{dx}(\arccos u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ with $u = x^2$: **M1**

$\frac{dy}{dx} = -\frac{2x}{\sqrt{1-x^4}}$ **A1**

Substitute $x = \frac{1}{\sqrt{2}}$, noting $x^4 = \frac{1}{4}$: $\frac{dy}{dx} = -\frac{\frac{2}{\sqrt{2}}}{\sqrt{\frac{3}{4}}}$ **M1**

$= -\frac{\sqrt{2}}{\frac{\sqrt{3}}{2}} = -\frac{2\sqrt{2}}{\sqrt{3}}$ **A1**

Gradient $= -\frac{2\sqrt{6}}{3}$ **A1**

Question 5 — 5 marks

Answer: 2

Substituting $x = 0$ gives the indeterminate form $\frac{0}{0}$ **R1**

Apply L'Hôpital's rule (differentiate numerator and denominator):

$\lim_{x\to 0}\frac{2e^{2x}-2}{2x}$ $M1$**A1**

This is again of the form $\frac{0}{0}$, so apply L'Hôpital's rule a second time:

$\lim_{x\to 0}\frac{4e^{2x}}{2}$ **A1**

Evaluate at $x = 0$: $\frac{4(1)}{2} = 2$ **A1**

Question 6 — 5 marks

Answer: $5/(48\pi)$

Find the radius when $V = 288\pi$:

$\frac{4}{3}\pi r^3 = 288\pi \Rightarrow r^3 = 216$ **(M1)**

$r = 6$ cm **A1**

Differentiate $V = \frac{4}{3}\pi r^3$:

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ **M1**

Substitute:

$15 = 4\pi(6)^2\frac{dr}{dt} = 144\pi\frac{dr}{dt}$ **A1**

$\frac{dr}{dt} = \frac{15}{144\pi} = \frac{5}{48\pi} \approx 0.0332\ \text{cm s}^{-1}$ **A1**

Question 7 — 5 marks

Answer: $y = -\frac{x}{12} + \frac{7}{4}$

$3y^2\frac{dy}{dx} + y + x\frac{dy}{dx} = 0$ $M1$**A1**

$\frac{dy}{dx} = -\frac{y}{3y^2+x} = -\frac{1}{12}$ **A1**

$y - 1 = -\frac{1}{12}(x - 9)$ **M1**

$y = -\frac{x}{12} + \frac{7}{4}$ **A1**

Question 8 — 5 marks

Answer: $(2, 4)$ and $(-2, -4)$

$\frac{dy}{dx} = \frac{y-2x}{2y-x}$ $M1$**A1**

horizontal: $y = 2x$ **A1**

$x^2 - 2x^2 + 4x^2 = 12$, $x = \pm 2$ **A1**

$(2, 4)$ and $(-2, -4)$ **A1**

Question 9 — 7 marks

Answer: $4/65$

Let h be the height of the rocket:

$\tan\theta = \frac{h}{2}$ $M1$**A1**

Differentiate with respect to t:

$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{2}\frac{dh}{dt}$ $M1$**A1**

When $h = 3$: $\tan\theta = \frac{3}{2}$, so $\sec^2\theta = 1 + \frac{9}{4} = \frac{13}{4}$ **A1**

Substitute:

$\frac{13}{4}\frac{d\theta}{dt} = \frac{1}{2}(0.4) = 0.2$ **(M1)**

$\frac{d\theta}{dt} = \frac{0.8}{13} = \frac{4}{65}\ \text{rad s}^{-1}$ **A1**

Question 10 — 6 marks

Answer: (a) $-\frac{2x+3y}{3x+2y}$; (b) $-\frac{8}{7}$

(a)

$2x + 3y + 3x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$M1A1$**A1**

$\frac{dy}{dx} = -\frac{2x+3y}{3x+2y}$ **A1**

(b)

$-\frac{2+6}{3+4}$ **M1**

$-\frac{8}{7}$ **A1**

Every solution is also on ibmathrevision.com/hlaa/advanced-differentiation-exam-practice
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