Substitution or integration by parts?

Name _____

Date _____

Definite integrals. Angles in radians. Give answers exactly or to 3 significant figures.

Part A: one of each

Each question tells you which method to use.

1 **SUBSTITUTION**

Find $\int_0^1 2x(x^2+1)^3\,dx$.

Answer: _____

2 **BY PARTS**

Find $\int_0^1 xe^x\,dx$.

Answer: _____

Part B: you decide

Substitution
The integrand contains a function and (a multiple of) its derivative, such as $x\,e^{x^2}$ or $\dfrac{2x}{x^2+1}$. Let u be the inside function.

By parts $\int u\dfrac{dv}{dx}\,dx = uv - \int v\dfrac{du}{dx}\,dx$
A product of two different kinds of function, such as $x\sin x$ or $x\ln x$, where one is not the derivative of the other.

For each integral, tick the method you need, then evaluate it. Each answer matches a letter in the code table. The letters from questions 1 to 8, in order, spell a word.

1

Find $\int_0^2 xe^{x^2}\,dx$.

Which method? ☐ Substitution ☐ By parts

Answer: _____ Letter: ___

2

Find $\int_1^e x\ln x\,dx$.

Which method? ☐ Substitution ☐ By parts

Answer: _____ Letter: ___

3

Find $\int_0^{\pi/2} \sin x\cos^3 x\,dx$.

Which method? ☐ Substitution ☐ By parts

Answer: _____ Letter: ___

4

Find $\int_0^{\pi} x\sin x\,dx$.

Which method? ☐ Substitution ☐ By parts

Answer: _____ Letter: ___

5

Find $\int_0^1 x^2 e^{x^3}\,dx$.

Which method? ☐ Substitution ☐ By parts

Answer: _____ Letter: ___

6

Find $\int_1^2 \ln x\,dx$. (There is only one function here.)

Which method? ☐ Substitution ☐ By parts

Answer: _____ Letter: ___

7

Find $\int_0^1 \frac{x}{x^2+1}\,dx$.

Which method? ☐ Substitution ☐ By parts

Answer: _____ Letter: ___

8

Find $\int_0^1 xe^{-x}\,dx$.

Which method? ☐ Substitution ☐ By parts

Answer: _____ Letter: ___

−3.14	−1	−0.25	0.25	0.264	0.347	0.386	0.573	0.693	1.39	1.72	2.10	3.14	5.29	26.8	53.6
Y	**V**	**M**	**T**	**L**	**A**	**R**	**G**	**H**	**Z**	**S**	**N**	**E**	**D**	**I**	**P**

1	2	3	4	5	6	7	8

Answers

Part A

1. Let $u = x^2 + 1$, $du = 2x\,dx$; $x = 0 \to u = 1$, $x = 1 \to u = 2$. $\displaystyle\int_1^2 u^3\,du = \left[\frac{u^4}{4}\right]_1^2 = \frac{15}{4} = 3.75$

2. $u = x$, $v' = e^x$: $[xe^x]_0^1 - \displaystyle\int_0^1 e^x\,dx = e - (e - 1) = 1$

Part B

1. Substitution. $u = x^2$: $\left[\frac{1}{2}e^{x^2}\right]_0^2 = \frac{1}{2}(e^4 - 1) = 26.8$ → **I**

2. By parts. $u = \ln x$, $v' = x$: $\left[\frac{x^2}{2}\ln x - \frac{x^2}{4}\right]_1^e = \frac{e^2+1}{4} = 2.10$ → **N**

3. Substitution. $u = \cos x$, $du = -\sin x\,dx$: $\left[-\frac{\cos^4 x}{4}\right]_0^{\pi/2} = \frac{1}{4} = 0.25$ → **T**

4. By parts. $u = x$, $v' = \sin x$: $[-x\cos x + \sin x]_0^\pi = \pi = 3.14$ → **E**

5. Substitution. $u = x^3$: $\left[\frac{1}{3}e^{x^3}\right]_0^1 = \frac{e-1}{3} = 0.573$ → **G**

6. By parts. Write $\ln x$ as $1 \times \ln x$; $u = \ln x$, $v' = 1$: $[x\ln x - x]_1^2 = 2\ln 2 - 1 = 0.386$ → **R**

7. Substitution. $u = x^2 + 1$: $\left[\frac{1}{2}\ln(x^2+1)\right]_0^1 = \frac{1}{2}\ln 2 = 0.347$ → **A**

8. By parts. $u = x$, $v' = e^{-x}$: $[-xe^{-x} - e^{-x}]_0^1 = 1 - \frac{2}{e} = 0.264$ → **L**

The word is **INTEGRAL**. Other letters come from common slips: forgetting the constant that the substitution brings in (questions 1, 5 and 7), sign errors, or leaving out the $-x$ when integrating $\ln x$ by parts in question 6.