Product, quotient or chain rule?

Differentiation. Angles in radians. Give answers exactly or to 3 significant figures.

Name _____

Date _____

Part A: one of each

Each question tells you which rule to use.

1 **PRODUCT RULE**

$y = x^2 e^x$. Find $\frac{dy}{dx}$ when $x = 1$.

Answer: _____

2 **QUOTIENT RULE**

$y = \frac{x}{x+1}$. Find $\frac{dy}{dx}$ when $x = 1$.

Answer: _____

3 **CHAIN RULE**

$y = (3x+1)^4$. Find $\frac{dy}{dx}$ when $x = 0$.

Answer: _____

Part B: you decide

Product $\frac{d}{dx}(uv) = u'v + uv'$
Two functions of x multiplied together.

Quotient $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$
One function of x divided by another.

Chain $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
A function of a function, such as $(\ldots)^n$, $e^{(\ldots)}$, $\ln(\ldots)$, $\sin(\ldots)$.

For each function, tick the rule you need, then find the derivative at the given value of x. Each answer matches a letter in the code table. The letters from questions 1 to 8, in order, spell a word.

1

$f(x) = x^3 \ln x$. Find $f'(2)$.

Which method? ☐ Product ☐ Quotient ☐ Chain

Answer: _____ Letter: ___

2

$f(x) = \sqrt{4x+1}$. Find $f'(2)$.

Which method? ☐ Product ☐ Quotient ☐ Chain

Answer: _____ Letter: ___

3

$f(x) = \dfrac{x^2}{x-3}$. Find $f'(1)$.

Which method? ☐ Product ☐ Quotient ☐ Chain

Answer: _____ Letter: ___

4

$f(x) = e^{x^2}$. Find $f'(1)$.

Which method? ☐ Product ☐ Quotient ☐ Chain

Answer: _____ Letter: ___

5

$f(x) = x \sin x$. Find $f'(1)$.

Which method? ☐ Product ☐ Quotient ☐ Chain

Answer: _____ Letter: ___

6

$f(x) = \dfrac{e^x}{x+2}$. Find $f'(1)$.

Which method? ☐ Product ☐ Quotient ☐ Chain

Answer: _____ Letter: ___

7

$f(x) = \ln(x^2+5)$. Find $f'(2)$.

Which method? ☐ Product ☐ Quotient ☐ Chain

Answer: _____ Letter: ___

8

$f(x) = x(2x-1)^3$. Find $f'(1)$.

Which method? ☐ Product ☐ Quotient ☐ Chain

Answer: _____ Letter: ___

−1.25	−0.604	0.111	0.167	0.444	0.540	0.604	0.667
A	**W**	**C**	**S**	**N**	**U**	**E**	**R**
1.25	1.38	2.72	4	5.44	6	7	12.3
B	**I**	**H**	**V**	**D**	**F**	**T**	**G**

1 2 3 4 5 6 7 8

Answers

Part A

1. $\dfrac{dy}{dx} = 2xe^x + x^2e^x$, so at $x = 1$: $3e = 8.15$

2. $\dfrac{dy}{dx} = \dfrac{(x+1)(1) - x(1)}{(x+1)^2} = \dfrac{1}{(x+1)^2}$, so at $x = 1$: $\frac{1}{4} = 0.250$

3. $\dfrac{dy}{dx} = 4(3x+1)^3 \times 3 = 12(3x+1)^3$, so at $x = 0$: 12

Part B

1. Product. $f'(x) = 3x^2 \ln x + x^2$, so $f'(2) = 12.3$ → **G**

2. Chain. $f'(x) = \dfrac{2}{\sqrt{4x+1}}$, so $f'(2) = 0.667$ → **R**

3. Quotient. $f'(x) = \dfrac{(x-3)(2x) - x^2}{(x-3)^2}$, so $f'(1) = -1.25$ → **A**

4. Chain. $f'(x) = 2xe^{x^2}$, so $f'(1) = 5.44$ → **D**

5. Product. $f'(x) = \sin x + x\cos x$, so $f'(1) = 1.38$ → **I**

6. Quotient. $f'(x) = \dfrac{(x+2)e^x - e^x}{(x+2)^2}$, so $f'(1) = 0.604$ → **E**

7. Chain. $f'(x) = \dfrac{2x}{x^2+5}$, so $f'(2) = 0.444$ → **N**

8. Product. The chain rule is needed for $(2x-1)^3$ inside it. $f'(x) = (2x-1)^3 + 6x(2x-1)^2$, so $f'(1) = 7$ → **T**

The word is **GRADIENT**. Other letters come from common slips: multiplying the two derivatives instead of using the product rule, writing the top of the quotient rule the wrong way round, or forgetting to multiply by the derivative of the inside.